# STAT 544 Applied Probability

Spring 2008

**Homework #0**

Due Wednesday 1/30/08

The purpose of this homework assignment is to make sure that you have the basic mathematical skills that are necessary for this course. This assignment will be graded, but the score will not count toward your course grade. You should work independently on this assignment, so you and I can assess your background and skills.

You should be able to do all of the problems. We will use all of the material at some point in the semester. If your skills are rusty because you have not used the concepts in a while, you will need to brush up on the material so that you are prepared when we need to use it.

Most of the problems in the math pretest include a reference to relevant chapters in two useful Schaum's outlines. These outlines contain concise topic summaries and many examples.

[SC] F. Ayres, Jr. and E. Mendelson, Schaum's Outline of Calculus, McGraw-Hill, 1999.

[SP] S. Lipschutz and M. Lipson, Schaum's Outline of Probability, 2nd Ed., McGraw-Hill, 2000.

1. [SP, Ch. 1]
   Let a universal set be given by $\mathcal{U} = \{1,2,3,4,5\}$. Subsets of $\mathcal{U}$ are $A = \{1,5\}$, $B = \{2,3,5\}$, $C = \{2,4\}$. Let the empty set be denoted by $\emptyset = \{\,\}$. Evaluate the following:

   (a) $A \cap (B \cup C)$ b. $(A \cap B) \cup (A \cap C^c)$ c. $A^c \cap (B^c \cap C^c)$ d. $(A \cap C)^c \cup B$

2. [SC, p. 206, Sigma Notation]
   Let $A$, $B$, and $C$ be constants. Evaluate the following sums:

   (a) $\sum_{x=1}^{6} C$ b. $\sum_{x=1}^{6} x$ c. $\sum_{x=1}^{6} x^2$ d. $\left(\sum_{x=1}^{6} x\right)^2$ e. $\sum_{x=1}^{6} \left(Ax^2 + Bx + C\right)$

3. [SC, p. 206, Sigma Notation]
   Let $C$ and $t$ be constants, and let $p(x)$ be defined for $x = 0, 1, 2$ as follows:

   | $x$ | 0 | 1 | 2 |
   |---|---|---|---|
   | $p(x)$ | 0.5 | 0.3 | 0.2 |

   . Evaluate the following sums:

   (a) $\sum_{x=0}^{2} x\,p(x)$ b. $\sum_{x=0}^{2} C\,p(x)$ c. $\sum_{x=0}^{2} x^2\,p(x)$ d. $\sum_{x=0}^{2} e^{tx}\,p(x)$

4. [This is just algebra]
   For each of the following functions, find $y$ as a function of $z$.

   (a) $z = -c\,\ln(y)$ b. $z = \dfrac{c}{\sqrt{y}}$ c. $z = \dfrac{y}{y+1}$

5. [SC, Ch. 43, 46]
   Let $a$ be a constant. The following infinite series converge and have a closed form solution. Find the solution. You don't have to "solve" these problems. Instead, you may look them up as identities in a calculus book or other reference.

   (a) $\sum_{x=0}^{\infty} a^x$ (For $|a| < 1$) b. $\sum_{x=0}^{\infty} \dfrac{a^x}{x!}$ (For all $a$)

6. [SC, Ch. 10, 17, 26, 48]
   Evaluate the following derivatives

   (a) $\dfrac{d}{dt}\,e^{xt}$ b. $\dfrac{d^2}{dt^2}\,e^{xt}$ c. $\dfrac{d}{dt}\left(0.5 + 0.3e^t + 0.2e^{2t}\right)$ d. $\dfrac{d}{dt}\,e^{-3t^2}$

   (b) $\dfrac{d}{dy}\,x\,\cos y$ f. $\dfrac{d}{dy}\,\arcsin y$ g. $\dfrac{d}{dy}\,\sqrt{4-y}$ h. $\dfrac{d}{dy}\,\dfrac{1}{y^2}$

7. [SC, Ch. 10, 26]
   For each of the following functions, find $f'(t) = \dfrac{d}{dt}f(t)$ and $f'(0)$.

   (a) $f(t) = \left(pe^t + 1 - p\right)^n$ b. $f(t) = \dfrac{1}{(1-bt)^a}$

8. [SC, Ch. 27]
Find $f(0)$ for $f(x) = x\ln(x)$ by expressing $f(x)$ as the fraction $f(x) = \frac{\ln(x)}{x^{-1}}$ and using L'Hospital's rule.

9. [SC, Ch. 14]
Find the absolute and relative maxima and minima of $f(x) = x^2(1-x)^2$ on the interval $0 \leq x \leq 1$.

10. [SC, Ch. 47]
Find the second order Taylor series approximation for $f(x) = e^{3x}$ at $x_o = 0$.

11. [SC, Ch. 3, 4]
Plot the following regions in the $xy$ plane

   (a) $\{(x,y) : 0 \leq x \leq 1,\ 0 \leq y \leq 1,\ 0 \leq x+y \leq 1\}$
   (b) $\{(x,y) : x \leq 0,\ y \geq 0,\ x^2+y^2 \leq 4\}$
   (c) $\{(x,y) : x \geq 0,\ y \geq 0,\ xy \geq 1,\ \frac{x}{y} \geq 1\}$

12. [SC, Ch. 22-26, 31, 35]
Evaluate the following integrals

   (a) $\int_0^2 x - \frac{x^2}{2} + x^3\, dx$ b. $\int_4^\infty e^{-x/2}\, dx$ c. $\int_0^\infty xe^{-2x}\, dx$ (Use integration by parts)

13. [SC, p. 210, Properties of the Definite Integral]
For any positive real number $\lambda$ and any positive integer $n$, we have the following integral identity: $\int_0^\infty \frac{\lambda e^{-\lambda x}(\lambda x)^n}{n!}\, dx = 1$. Using this identity, evaluate $\int_0^\infty x^5 e^{-2x}\, dx$.

14. [SC, Ch. 54]
Evaluate the following double integrals

   (a) $\int_1^2 \int_0^{0.5} (x+y)\, dx\, dy$ b. $\int_2^3 \int_0^x xy\, dy\, dx$

15. [SC, Ch. 41, 55]
Compute the following double integral by first converting it into an equivalent integral in polar coordinates.

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$$